

Process capability is the long-term performance level of the process after it has been brought under statistical control. It is the ability of the combination of your 5 M's to produce a product that will consistently meet the design requirements and the customer expectation.

In this article, Dr. Nosh Kapadia explains: How to calculate Cp, Cpk, Cpl and Cpu indices, Why should You know the Capability of Your and your supplier's Processes, The difference between Capability indices and Process performance indices, The relation between Process capability and defect rate and Some assumptions and precautions while calculating process capability indices.

## Preamble:

This article is devoted to the topic of process capability, with the objective of making people aware of this subject and its significance to business success.

The author believes that personal awareness is a prerequisite to personal action, and personal action is what we need for success.

- It can be a source material for you to use in discussing this topic with your organization.
- It will address issues like what is process capability, how to measure it, and how to calculate the process capability indices (Cp, Cpk).
- It will also attempt to explain the differences between process capability and process performance; relationship between Cpk and non-conforming (defect) rate; and illustrate the four outcomes of comparing natural process variability with customer specifications.
- Lastly a commentary is provided on precautions we should take while conducting process capability studies.

## What is Process Capability?

1. Process capability is the long-term performance level of the process after it has been brought under statistical control. In other words, process capability is the range over which the natural variation of the process occurs as determined by the system of common causes.
2. Process capability is also the ability of the combination of people, machine, methods, material, and measurements to produce a product that will consistently meet the design requirements or customer expectation.

## What is a Process Capability Study?

Process capability study is a scientific and a systematic procedure that uses control charts to detect and eliminate the unnatural causes of variation until a state of statistical control is reached. When the study is completed, you will identify the natural variability of the process.

## Why Should I know the Capability of My Processes?

- Process capability measurements allow us to summarize process capability in terms of meaningful percentages and metrics.
- To predict the extent to which the process will be able to hold tolerance or customer requirements. Based on the law of probability, you can compute how often the process will meet the specification or the expectation of your customer.
- You may learn that bringing your process under statistical control requires fundamental changes - even redesigning and implementing a new process that eliminates the sources of variability now at work.
- It helps you choose from among competing processes, the most appropriate one for meeting customers' expectation.
- Knowing the capability of your processes, you can specify better the quality performance requirements for new machines, parts and processes.

## Why Should I know the Capability of My Supplier's Processes ?

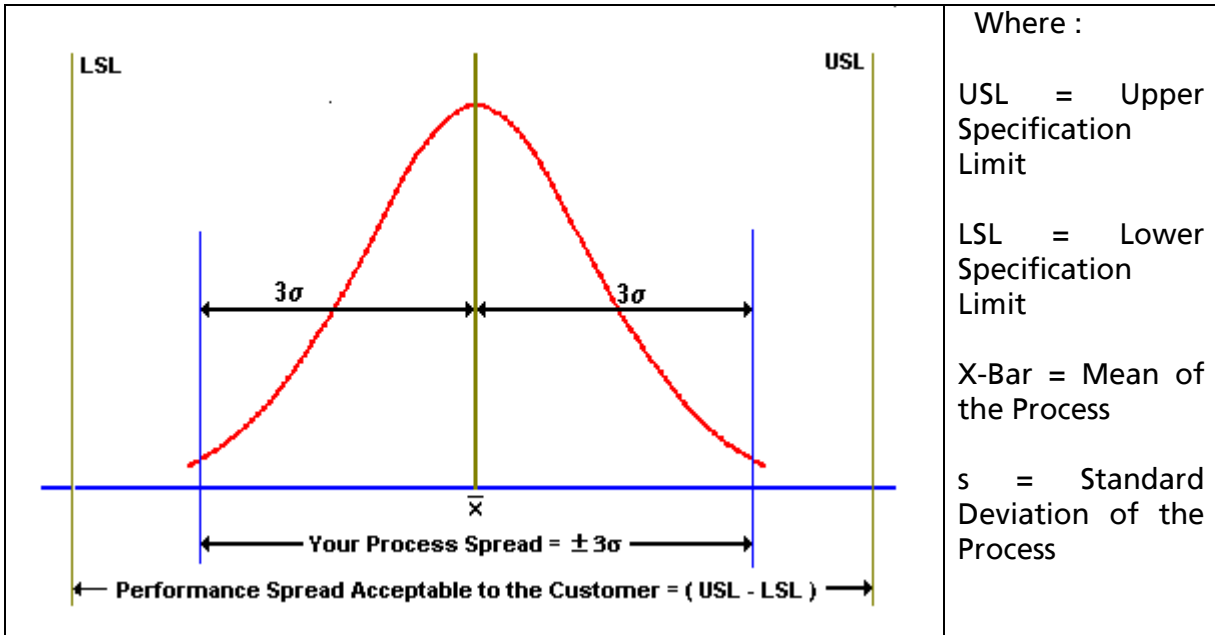
1. To set realistic cost effective part specifications based upon the customer's needs and the costs associated by the supplier at meeting those needs.
2. To understand hidden supplier costs. Suppliers may not know or hide their natural capability limits in an effort to keep business. This could mean that unnecessary costs could occur such as sorting to actually meet customer needs.
3. To be pro-active. For example, a Cpk estimation made using injection molding pressure measurements during a molding cycle may help reveal a faulty piston pressure valve ready to malfunction before the actual molded part measurements go out of specifications. Thus saving time and money.

## Measures of Process Capability - Process Capability Indices:

Cp, Cpl, Cpu, and Cpk are the four most common and timed tested measures of process capability.

- Process capability indices measure the degree to which your process produces output that meets the customer's specification.
- Process capability indices can be used effectively to summarize process capability information in a convenient unitless system.
- Cp and Cpk are quantitative expressions that personify the variability of your process (its natural limits) relative to its specification limits (customer requirements).

Following are the graphical details and equations quantifying process capability:



INDEX	ESTIMATED EQUATION	USAGE
Cp	$(USL - LSL) / 6s$	Process Capability for two - sided specification limit, irrespective of process center.
Cpu	$(USL - X-Bar) / 3s$	Process Capability relative to upper specification limit.
Cpl	$(X-Bar - LSL) / 3s$	Process Capability relative to lower specification limit.
Cpk	Min. of (Cpu , Cpl ) or Distance between mean of the process and the closest spec. limit / 0.5 of the process variability.	Process Capability for two - sided specification limit accounting for process centering.

**Notes :**

1. If X-Bar is at target, then Cp = Cpk.
2. Cpk will always be equal to or less than Cp.

**The Cpk, Ppk Quandary :**

In 1991, ASQ / AIAG task force published the "Statistical Process Control" reference manual, which presented the calculations for capability indices ( Cp, Cpk ) as well as process performance indices ( Pp, Ppk ).

The difference between the two indices is the way the process standard deviation (  $s$  ) is calculated.

**Cpk uses  $s$  which is estimated using (  $R\text{-Bar} / d_2$  ) or (  $S\text{-Bar} / C_2$  ) .**

**Ppk uses the calculated standard deviation from individual data where  $s$  is calculated by the formula :**

$$\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

So the next question is which metric is best to report Cpk or Ppk?

In other words, which standard deviation to use - estimated or calculated?

Although both indices show similar information, they have slightly different uses.

- Ppk attempts to answer the question "does my current production sample meet specification?" Process performance indices should only be used when statistical control cannot be evaluated.
- On the other hand, Cpk attempts to answer the question "does my process in the long run meet specification?" Process capability evaluation can only be done after the process is brought into statistical control. The reason is simple: Cpk is a prediction, and one can only predict something that is stable.

**The readers should note that Ppk and Cpk indices would likely be similar when the process is in a state of statistical control.**

### Notes:

1. As a thumb rule a minimum of 50 randomly selected samples must be chosen for process performance studies and a minimum of 20 subgroups (of sample size, preferably of at least 4 or 5) must be chosen for process capability studies.
2. Cpk for all critical product measurements considered important by the customer should be calculated at the beginning of initial production to determine the general ability of the process to meet customer specifications. Then from time to time, over the life of the product, Cpk's must be generated. A control chart must always be maintained to check statistical stability of the process before capability is computed.

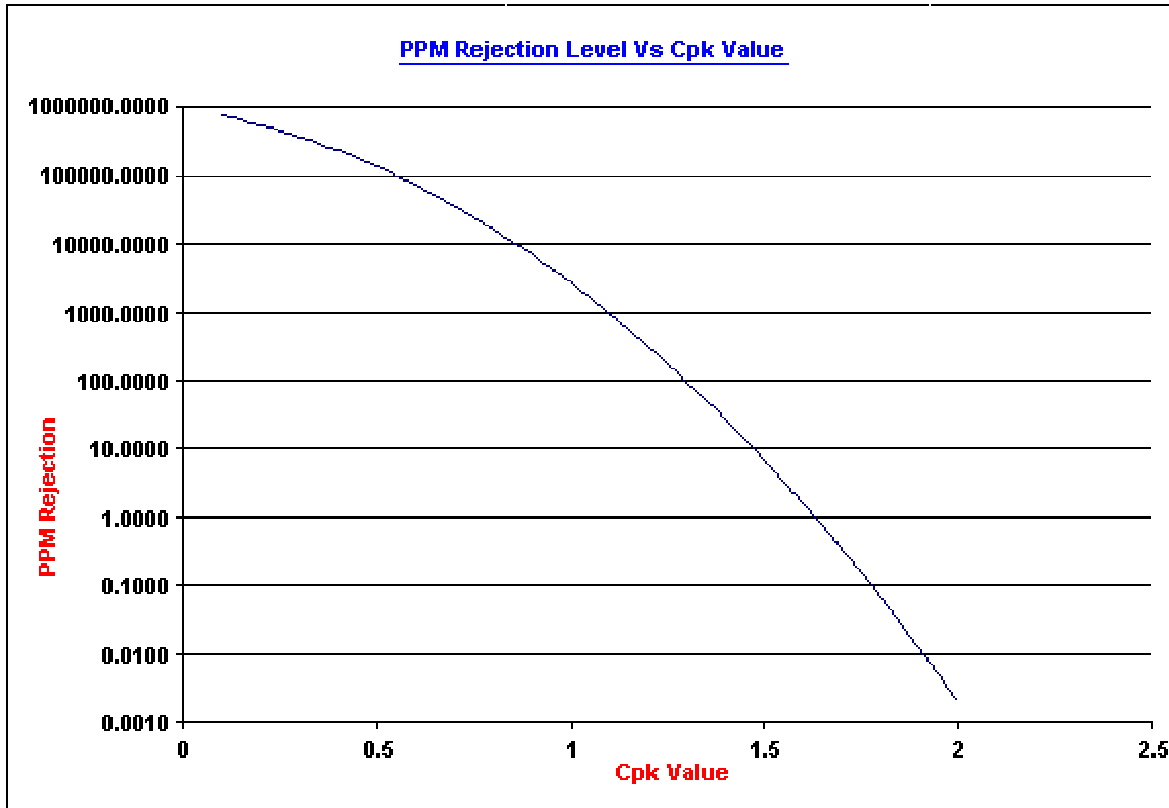
## Process Capability and Defect Rate :

Using process capability indices it is easy to forget how much of product is falling beyond specification. The conversion curve presented here can be a useful tool for interpreting Cpk with its corresponding defect levels. The defect levels or parts per million non-conforming were computed for different Cpk values using the Z scores and the percentage area under the standard normal curve using normal deviate tables.

The table below presents the non-conforming parts per million (ppm) for a process corresponding to Cpk values if the process mean were at target.

Cpk Value	Sigma Value	Area under Normal Curve	Non Conforming ppm
0.1	0.3	0.235822715	764177.2851
0.2	0.6	0.451493870	548506.1299
0.3	0.9	0.631879817	368120.1835
0.4	1.2	0.769860537	230139.4634
0.5	1.5	0.866385542	133614.4576
0.6	1.8	0.928139469	71860.531
0.7	2.1	0.964271285	35728.7148
0.8	2.4	0.983604942	16395.0577
0.9	2.7	0.993065954	6934.0461
1.0	3.0	0.997300066	2699.9344
1.1	3.3	0.999033035	966.9651
1.2	3.6	0.999681709	318.2914
1.3	3.9	0.999903769	96.231
1.333	3.999	0.999936360	63.6403
1.4	4.2	0.999973292	26.7082
1.5	4.5	0.999993198	6.8016
1.6	4.8	0.999998411	1.5887
1.666	4.998	0.999999420	0.5802
1.7	5.1	0.999999660	0.3402
1.8	5.4	0.999999933	0.0668
1.9	5.7	0.999999988	0.012
2.0	6.0	0.999999998	0.002

The Cpk conversion curve for process with mean at target is shown next.



Explanation: A process with Cpk of 2.0 (+/- 6 sigma capability), i.e., the process mean is 6 sigma away from the nearest specification can be expected to have no more than 0.002 nonconforming parts per million.

This process is so good that even if the process mean shifts by as much as +/- 1.5 sigma the process will produce no more than 3.4 non-conforming parts per million.

The next section provides the reader with some practical clarifications on Process Capability (Voice of the process) and Specification (Expectations of the customer).

## Natural Variability versus Specifications for Process Capability:

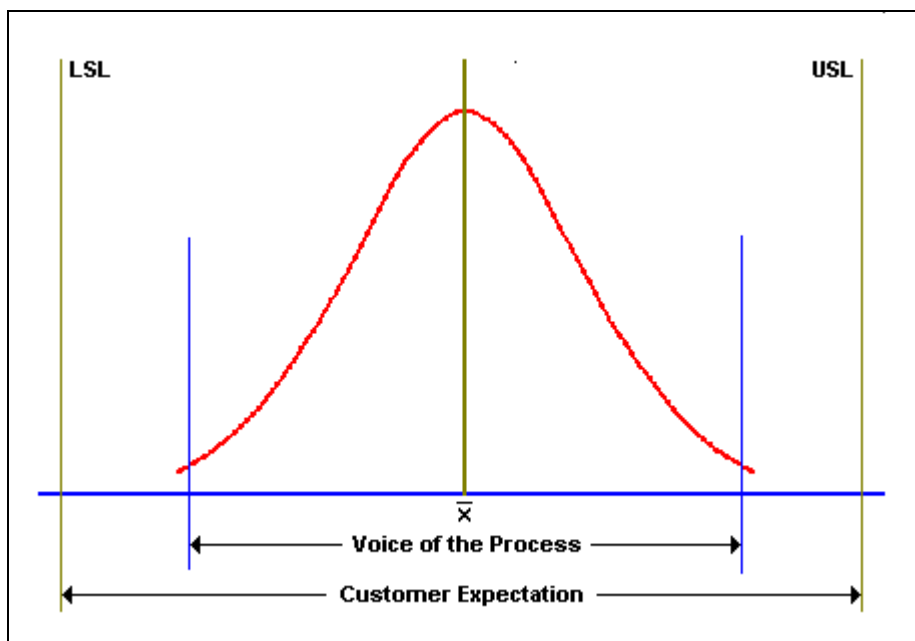
As seen from the earlier discussions, there are three components of process capability:

1. Design specification or customer expectation (Upper Specification Limit, Lower Specification Limit)
2. The centering of the natural process variation (X-Bar)
3. Spread of the process variation ( $s$ )

A minimum of four possible outcomes can arise when the natural process variability is compared with the design specifications or customer expectations:

### Case 1: $C_{pk} > 1.33$ (A Highly Capable Process)

This process should produce less than 64 non-conforming ppm



A Highly Capable Process: Voice of the Process < Specification ( or Customer Expectations ).

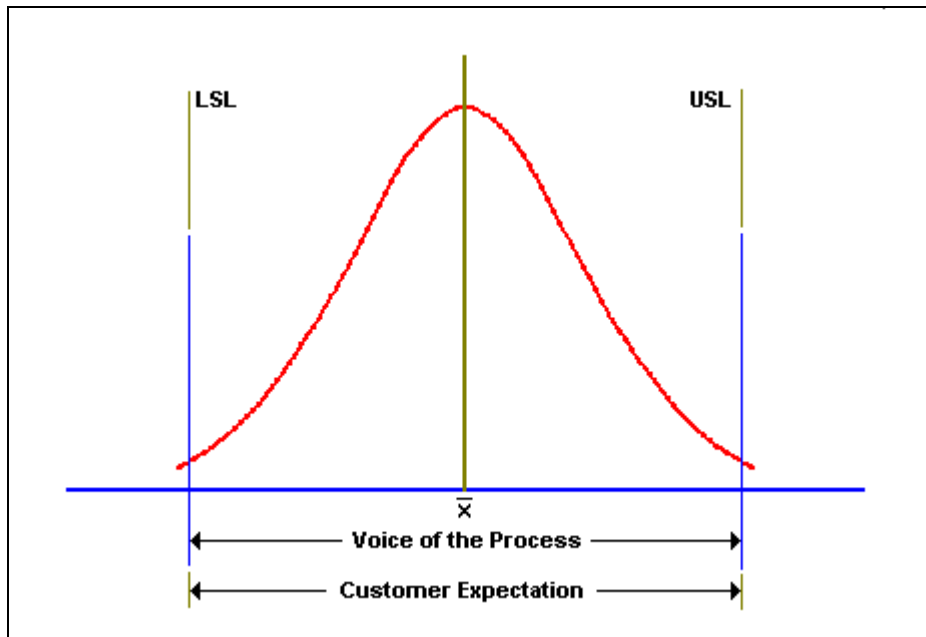
This process will produce conforming products as long as it remains in statistical control. The process owner can claim that the customer should experience least difficulty and greater reliability with this product. This should translate into higher profits.

**Note:**  $C_{pk}$  values of 1.33 or greater are considered to be industry benchmarks. This means that the process is contained within four standard deviations of the process specifications.

## Case 2: Cpk = 1 to 1.33 (A Barely Capable Process)

This process will produce greater than 64 ppm but less than 2700 non-conforming ppm.

This process has a spread just about equal to specification width. It should be noted that if the process mean moves to the left or the right, a significant portion of product will start falling outside one of the specification limits. This process must be closely monitored.



A Barely Capable Process: Voice of the Process = Customer Expectations

**Note:** This process is contained within three to four standard deviations of the process specifications.



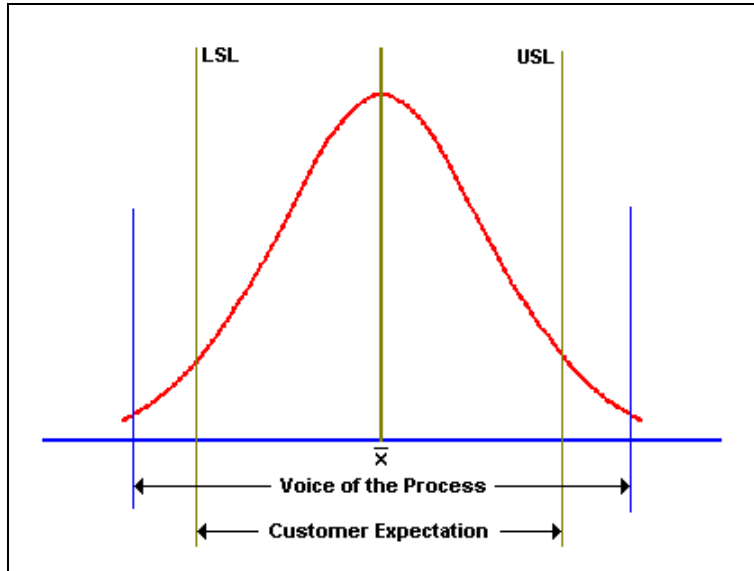
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- Anderson-Darling Normality test with user-defined p value
- Normality statistic: Skewness and Kurtosis

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### Case 3: $C_{pk} < 1$ (The Process is not Capable)

This process will produce more than 2700 non-conforming ppm.

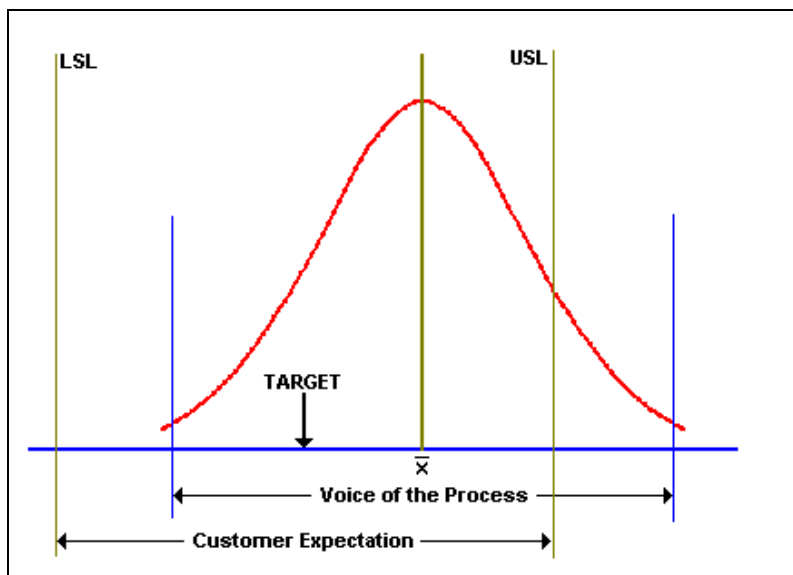


A Non-Capable Process: Voice of the Process > Customer Expectations.

It is impossible for the current process to meet specifications even when it is in statistical control. If the specifications are realistic, an effort must be immediately made to improve the process (i.e. reduce variation) to the point where it is capable of producing consistently within specifications.

### Case 4: $C_{pk} < 1$ ( The Process is not Capable )

This process will also produce more than 2700 non-conforming ppm.



The variability ( $s$ ) and specification width is assumed to be the same as in case 2, but the process average is off-center. In such cases, adjustment is required to move the process mean back to target. If no action is taken, a substantial portion of the output will fall outside the specification limit even though the process might be in statistical control.

### Assumptions, Conditions and Precautions :

Capability indices described here strive to represent with a single number the capability of a process. Much has been written in the literature about the pitfalls of these estimates. Following are some of the precautions the readers should exercise while calculating and interpreting process capability:

1. The indices for process capability discussed are based on the assumption that the underlying process distribution is approximately bell shaped or normal. Yet in some situations the underlying process distribution may not be normal. For example, flatness, pull strength, waiting time, etc., might naturally follow a skewed distribution. For these cases, calculating  $C_{pk}$  the usual way might be misleading. Many researchers have contributed to this problem. Readers are requested to refer to John Clements article titled "Process Capability Calculations for Non-Normal Distributions" for details.
2. The process / parameter in question must be in statistical control. It is this author's experience that there is tendency to want to know the capability of the process before statistical control is established. The presence of special causes of variation make the prediction of process capability difficult and the meaning of  $C_{pk}$  unclear.
3. The data chosen for process capability study should attempt to encompass all natural variations.  
For example, one supplier might report a very good process capability value using only ten samples produced on one day, while another supplier of the same commodity might report a somewhat lesser process capability number using data from longer period of time that more closely represent the process. If one were to compare these process index numbers when choosing a supplier, the best supplier might not be chosen.
4. The number of samples used has a significant influence on the accuracy of the  $C_{pk}$  estimate.

For example, for a random sample of size  $n = 100$  drawn from a know normal population of  $C_{pk} = 1$ , the  $C_{pk}$  estimate can vary from 0.85 to 1.15 ( with 95 % confidence ). Therefore smaller samples will result in even larger variations of the  $C_{pk}$  statistics. In other words, the practitioner must take into consideration the sampling variation's influence on the computed  $C_{pk}$  number. Please refer to Bissell and Chou, Owen, and Borrego for more on this subject.

### Concluding Thoughts:

In the real world, very few processes completely satisfy all the conditions and assumptions required for estimating Cpk. Also, statistical debates in research communities are still raging on the strengths and weaknesses of various capability and performance indices. Many new complicated capability indices have also been invented and cited in literature. However, the key to effectual use of process capability measures continues to be the level of user understanding of what these measures really represent. Finally, in order to achieve continuous improvement, one must always attempt to refine the "Voice of the Process" to match and then to surpass the "Expectations of the Customer".

### References:

1. Victor Kane, "Process Capability Indices", Journal of Quality Technology, Jan 1986.
2. ASQ / AIAG, "Statistical Process Control", Reference Manual, 1995.
3. John Clements, "Process Capability Calculations for Non-Normal Distributions", Quality Progress, Sept 1989.
4. Forrest Breyfogle, "Measurement of Process Capability", Smarter Solutions, 1996.
5. Bissell, "How Reliable is Your Capability Index", Royal Statistical Society, 1990.
6. Chou, Owen, and Borrego, "Lower Confidence Limits of Process Capability Indices", Journal of Quality Technology, Vol 22, No. 3, July 1990.

### Author:

Dr. Mehernosh Kapadia has a Master's Degree in Mechanical Engineering from Penn State University, USA and MBA from NTU, USA. He is a Six-Sigma Master Black Belt and an ASQ certified CQE & CRE. He obtained his Doctorate from The University of Bombay.

He can be contacted through us at e-mail address: <mailto:editor@symphonytech.com>

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### Non-Normal Process Capability Evaluator

Process capability evaluation done the traditional way, is based on an assumption that the data is Normally distributed. However, there are processes, which are inherently non-normal, and using assumptions of normality can lead to erroneous evaluation and wrong decisions. In such cases, the evaluation of Process Capability requires special techniques. Recognizing this, the recently updated SPC manual of AIAG (2nd Edition, July 2005) recommends the use of Non-normal process capability evaluation techniques to deal with such data.

Symphony's Non-Normal Process Capability Evaluator, enables you evaluate non-normal process capability using:

- Box-Cox power Transformation
- Johnson Transformation system
- Clements Method using Pearson Curves

#### **Box-Cox Transformation:**

The Box-Cox transformation is used for transforming non-normal data. The Box-Cox Transformation uses the parameter Lambda. Non-Normal Process Capability Evaluator automatically selects the value of lambda so that the best possible transformation is done and data transforms as closely as possible to normality. Process Capability evaluation is done on the transformed data.

#### **Johnson Transformation:**

Johnson transformation internally evaluates several transform functions and optimally selects one from three families of distributions, which transform the data into a normal distribution. These distributions are bounded, lognormal, and unbounded (SB, SL & SU). Non-Normal Process Capability Evaluator selects the transform function such that the transformed data comes closest to Normality. Process Capability evaluation is done on the transformed data.

#### **Clement's Method using Pearson Curves:**

Clements' method estimates capability indices based on evaluation of the skewness and kurtosis of the data. The method estimates percentiles and medians of the process distribution to define percentile based performance indices. Clements (1989) proposed this method of determining percentiles based on Pearson family of distribution.

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**Additional Reading:** [Process Capability Evaluation - Looking beyond the obvious](http://www.symphonytech.com/articles/proccap2.htm)  
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**Symphony Technologies Pvt Ltd**

B/4, Saket, Vidnyan Nagar, Bawdhan,  
Pune 411 021, India.  
Tel: 91-20-2295 1276  
Fax: 91-20-2295 2158  
Email: mail@symphonytech.com  
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